

Abstract

This thesis presents new techniques for the construction and specification of linear tail-biting trellises. Tail-biting trellises for linear block codes are combinatorial descriptions in the form of layered graphs, that are somewhat more compact than the corresponding conventional trellis descriptions. Conventional trellises for block codes have a well understood underlying theory. On the other hand, the theory of tail-biting trellises appears to be somewhat more involved, though several advances in the understanding of the structure and properties of such trellises have been made in recent years. Of fundamental importance is the observation that a linear tail-biting trellis for a block code corresponds to a coset decomposition of the code with respect to a subcode. All constructions seem to use this property in some way, in other words, this is the unifying factor in all our constructions. The constructions yield the conventional trellis when the subcode is the whole code. We list the main contributions of this thesis

- (i) We generalize the well known Bahl-Cocke-Jelinek-Raviv construction for conventional trellises to obtain tail-biting trellises. We show that a linear tail-biting trellis for a linear block code C with block length n and dimension k over a finite field \mathbb{F}_q , can be constructed by specifying an arbitrary parity check matrix \mathcal{H} for C along with a *displacement matrix* $\mathcal{D} \in \mathbb{F}_q^{(n-k) \times k}$
- (ii) We show that the displacement matrix \mathcal{D} yields a coset decomposition of the code. We present a dynamic algorithm that starts with a generator matrix for the code C in trellis-oriented form, and computes the displacement matrix \mathcal{D} for a minimal trellis in time $\mathcal{O}(n^k)$
- (iii) Forney has given an elegant algebraic characterization of a conventional trellis in terms of a quotient group of the code with respect to a special subgroup. Given a coset decomposition of the code, we propose a natural extension for tail-biting trellises, which considers a circular time axis instead of a linear one. The quotient group of interest is formed by judiciously using the properties of coset leaders
- (iv) There is an interesting connection between algebraic and combinatorial duality. For conventional trellises, it is known that the primal and dual trellises have the same

state-complexity profile. We give a simple and direct construction for a dual trellis in terms of the generator matrix \mathcal{G} , the parity check matrix \mathcal{H} and the displacement matrix \mathcal{D} , and prove that the same property is true for tail-biting trellises.

- (v) We give a new characterization of linear tail-biting trellises in terms of an equivalence relation defined on a language derived from the code under consideration. The equivalence relation is intimately tied up with the coset decomposition of the code with respect to a subcode. In the context of formal language theory, this interpretation is an adaptation of the Myhill-Nerode theorem for regular languages.